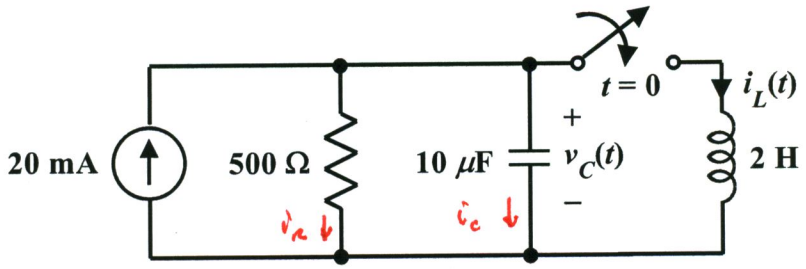
EE 2240

Problem #05

The circuit has reached the DC steady state prior to $t = 0$. Find $i_L(t)$ for $t > 0$.



$v_c(0) = (20mA)(500\Omega) = 10\ V$, $\quad i_L(0) = 0$

For $t \geq 0$:

$$v_c(t) = 2\frac{di_L}{dt} \Rightarrow \left.\frac{di_L}{dt}\right|_{t=0} = \frac{1}{2}v_c(0) = 5$$

$$i_c(t) = (10\mu F)\,\dot{v}_c(t) = 20\times10^{-6}\frac{d^2i_L}{dt^2}$$

$$i_R(t) = \frac{v_c(t)}{500\Omega} = 0.004\frac{di_L}{dt}$$

$$i_c + i_R + i_L = 0 \Rightarrow 20\times10^{-6}\frac{d^2i_L}{dt^2} + 0.004\frac{di_L}{dt} + i_L = 0.02$$

$$\Rightarrow \frac{d^2i_L}{dt^2} + 200\frac{di_L}{dt} + 50000\,i_L = 1000$$

$$r^2 + 200r + 50000 = (r+100)^2 + (200)^2$$

$$\Rightarrow r = -100 \pm j200$$

$$\therefore i_L(t) = e^{-100t}(K_1\cos 200t + K_2\sin 200t) + K_3$$

$$i_L(\infty) = 20\,mA \Rightarrow K_3 = 0.02$$

$$i_L(t) = e^{-100t}(K_1\cos 200t + K_2\sin 200t) + 0.02$$

$$\frac{di_L}{dt} = -100e^{-100t}(K_1\cos 200t + K_2\sin 200t) + e^{-100t}(-200K_1\sin 200t + 200K_2\cos 200t)$$

$$i_L(0) = K_1 + 0.02 = 0 \Rightarrow K_1 = -0.02$$

$$\left.\frac{di_L}{dt}\right|_{t=0} = -100K_1 + 200K_2 = 5 \Rightarrow K_2 = 0.015$$

$$\therefore i_L(t) = e^{-100t}(-0.02\cos 200t + 0.015\sin 200t) + 0.02\ A, \quad t \geq 0$$